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Sufficient optimality conditions and duality in multiobjective fractional programming involving generalized d-type-I functions

by
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Abstract

For a nonlinear fractional multiobjective programming problem some sufficient optimality conditions and duality results under generalized d-type-I assumptions are established.

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1 Introduction

We consider the following multiobjective nonlinear fractional programming problem:

$$(VFP)\quad \begin{array}{l} \min \frac{f(x)}{g(x)} = \left(\dfrac{f_1(x)}{g_1(x)}, \dots, \dfrac{f_p(x)}{g_p(x)}\right) \\ \text{subject to } \begin{cases} h_j(x) \leqq 0, \ j = 1, 2, \dots, m \\ x \in X_0, \end{cases} \end{array}$$

where $X_0 \subseteq \mathbf{R}^n$ is a nonempty open set, $f = (f_1, \dots, f_p)$, $g = (g_1, \dots, g_p) : X_0 \to \mathbf{R}^p$, $h = (h_1, \dots, h_m) : X_0 \to \mathbf{R}^m$, $g_i(x) > 0$ for all $x \in X_0$ and each $i = 1, \dots, p$. We denote $X = \{x \in X_0 \mid h_j(x) \leqq 0, \ j = 1, 2, \dots, m\}$, the feasible set of problem (VFP).

Optimality conditions and duality for nonlinear singleobjective or multiobjective optimization problems involving generalized convex functions have been of much interest in the recent past and many contributions have been made to this development, e.g. Antczak [1], Corley [2], Egudo [3], Geoffrion [5], Mishra [10], Mititelu [16], Mukherjee and Mishra [17].

There exists an equivallence between saddle-points of the Lagrangian and optima for an inequality constrained minimization problem, under a convexity assumption and a regularity hypothesis. In [7], Jeyakumar discussed a class of nonsmooth nonconvex problems in which functions are locally Lipschitz and are satisfying some invex type conditions and he proved that duality theorems of Wolfe type hold for this class of problems.

Mishra and Mukherjee extended the concept of V–invex functions to nonsmooth case [13] and nonsmooth composite case [9] and [11]. Mishra [8] and Mishra and Mukherjee [12] have also extended the class of V-invex functions to the case of continuous-time and established duality results for variational and control problems.

Preda and Stancu-Minasian [22] gave optimality conditions for weak vector minima using η-semidifferentials and functions satisfying generalized semilocally preinvex properties and used these results to extend the Wolfe and Mond-Weir duals, generalizing results of Preda [19], Preda et al. [23].

Preda [20] considered necessary and sufficient optimality conditions for a nonlinear fractional multiple objective programming problem involving η-semidifferentiable functions. Also, a general dual was formulated and duality results were proved using concepts of generalized semilocally preinvex functions. Thus, results of Preda [19], Preda et al. [23], Preda and Stancu-Minasian [22] were generalized.

Mishra et al. [15] extended the issues of Preda [20] to the case of semilocally type I and related functions, generalizing results of Preda [20].

Niculescu [18] defined $\alpha\eta$-locally starshaped sets, considered optimality conditions for (VFP) involving η-semidifferentiable functions and proved a duality result using generalized ρ-semilocally type I and related functions, extending the work of Mishra et al. [15].

In this paper, we obtain sufficient optimality conditions and a duality result for (VFP) involving more general classes of functions. Here, as in Preda [21], the place of the derivative is taken by a bifunction having certain properties. Thus, we extend the work of Niculescu [18] and generalize results obtained in the literature on this topic.

2 Definitions and Preliminaries

Throughout this paper we use the following conventions for x, $y \in \mathbf{R}^n$:

$$\begin{aligned} x &< y \text{ iff } x_i < y_i \text{ for any } i = \overline{1,n}; \\ x &\leqq y \text{ iff } x_i \leqq y_i \text{ for any } i = \overline{1,n}; \\ x &\leq y \text{ iff } x \leqq y \text{ and } x \neq y. \end{aligned}$$

$x \nless y$ is the negation of $x < y$.

We denote $\mathbf{R}^n_+ = \{x \in \mathbf{R}^n | x \geqq 0\}$.

Definition 2.1. *A point $\overline{x} \in X$ is said to be a weak Pareto solution or weak minimum for (VFP) if* $\frac{f(x)}{g(x)} \nless \frac{f(\overline{x})}{g(\overline{x})}$ *for all $x \in X$.*

Definition 2.2. *A point $\overline{x} \in X$ is said to be a local weak Pareto solution or local weak minimum for (VFP) if there is a neighborhood $V(\overline{x})$ around $\overline{x}$, such that*

$$\frac{f(x)}{g(x)} \not< \frac{f(\overline{x})}{g(\overline{x})} \text{ for all } x \in X \cap V(\overline{x}).$$

Let $\widehat{f}_i, \widehat{g}_i : X_0 \times \mathbf{R}^n \to \mathbf{R}$ for $i \in P = \{1, 2, ..., p\}$, and $\widehat{h}_j : X_0 \times \mathbf{R}^n \to \mathbf{R}$ for $j \in M = \{1, 2, ..., m\}$.

Let $\rho^1 = (\rho^1_1, ..., \rho^1_p)^T, \rho^2 = (\rho^2_1, ..., \rho^2_p)^T \in \mathbf{R}^p, \rho^3 = (\rho^3_1, ..., \rho^3_m) \in \mathbf{R}^m$, and $d : X_0 \times X_0 \to \mathbf{R}_+$.

Definition 2.3. *(see [21]).* *(f, g, h) is said to be of general $(\rho^1, \rho^2, \rho^3, d)$ - type I at $\overline{x} \in X_0$ relative to $\left(\widehat{f}_i, \widehat{g}_i, \widehat{h}_j\right), i \in P, j \in M$ if there exist the functions $\eta : X_0 \times X_0 \to \mathbf{R}^n, \alpha_i, \beta_i, \gamma_j : X_0 \times X_0 \to \mathbf{R}_+ \setminus \{0\}, i \in P, j \in M$ such that for all $x \in X_0$, we have*

$f_i(x) - f_i(\overline{x}) \geqq \alpha_i(x, \overline{x}) \widehat{f}_i(\overline{x}, \eta(x, \overline{x})) + \rho^1_i d(x, \overline{x}), \forall i \in P,$

$g_i(x) - g_i(\overline{x}) \leqq \beta_i(x, \overline{x}) \widehat{g}_i(\overline{x}, \eta(x, \overline{x})) - \rho^2_i d(x, \overline{x}), \forall i \in P,$

$-h_j(\overline{x}) \geqq \gamma_j(x, \overline{x}) \widehat{h}_j(\overline{x}, \eta(x, \overline{x})) + \rho^3_j d(x, \overline{x}), \forall j \in M.$

3 Sufficient Optimality Criteria

In this section we obtain some sufficient conditions for a feasible solution $\overline{x}$ to be weak minimum for (VFP).

Theorem 3.1. *Let $\overline{x} \in X$ such that $f(\overline{x}) \geqq 0$, and (f, g, h) be of general $(\rho^1, \rho^2, \rho^3, d)$ - type I at $\overline{x}$. Also, we assume that there exists $\lambda^0 \in \mathbf{R}^p$, $v^0 \in \mathbf{R}^m$ such that $\rho^2 \geqq 0$, $\sum_{i=1}^{p} \frac{\lambda^0_i \rho^1_i}{\alpha_i(x,\overline{x})} + \sum_{j=1}^{m} \frac{v^0_j \rho^3_j}{\gamma_j(x,\overline{x})} \geqq 0, \forall x \in X,$ and*

$$\sum_{i=1}^{p} \lambda^0_i \widehat{f}_i(\overline{x}, \eta(x, \overline{x})) + \sum_{j=1}^{m} v^0_j \widehat{h}_j(\overline{x}, \eta(x, \overline{x})) \geqq 0, \ \forall\, x \in X, \tag{3.1}$$

$$\widehat{g}_i(\overline{x}, \eta(x, \overline{x})) \leqq 0, \ \forall x \in X, \ \forall i \in P, \tag{3.2}$$

$$v^{0^T} h(\overline{x}) = 0, \tag{3.3}$$

$$\lambda^0 \geq 0, \tag{3.4}$$

$$v^0 \geqq 0. \tag{3.5}$$

Then $\overline{x}$ is a weak minimum solution for (VFP).

Proof: We proceed by contradicting. We assume that there exists $\widetilde{x} \in X$ such that

$$\frac{f_i(\widetilde{x})}{g_i(\widetilde{x})} < \frac{f_i(\overline{x})}{g_i(\overline{x})} \text{ for any } i \in P. \tag{3.6}$$

Since (f,g,h) is of general $\left(\rho^1,\rho^2,\rho^3,d\right)$ - type I at $\overline{x}$, we get

$$\frac{1}{\alpha_i(\widetilde{x},\overline{x})}\left(f_i(\widetilde{x})-f_i(\overline{x})\right) \geqq \widehat{f}_i(\overline{x},\eta(\widetilde{x},\overline{x}))+\frac{\rho_i^1}{\alpha_i(\widetilde{x},\overline{x})}d(\widetilde{x},\overline{x}),\forall\ i\in P, \tag{3.7}$$

$$\frac{1}{\beta_i(\widetilde{x},\overline{x})}\left(g_i(\widetilde{x})-g_i(\overline{x})\right) \leqq \widehat{g}_i(\overline{x},\eta(\widetilde{x},\overline{x}))-\frac{\rho_i^2}{\beta_i(\widetilde{x},\overline{x})}d(\widetilde{x},\overline{x}),\forall\ i\in P, \tag{3.8}$$

$$-\frac{h_j(\overline{x})}{\gamma_j(\widetilde{x},\overline{x})} \geqq \widehat{h}_j(\overline{x},\eta(\widetilde{x},\overline{x}))+\frac{\rho_j^3}{\gamma_j(\widetilde{x},\overline{x})}d(\widetilde{x},\overline{x}),\ \forall j\in M. \tag{3.9}$$

Multiplying (3.7) by $\lambda_i^0 \geqq 0$, $i\in P$, (3.10) by $v_j^0 \geqq 0$, $j\in M$ and then summing the obtained relations, we get

$$\sum_{i=1}^{p}\frac{\lambda_i^0}{\alpha_i(\widetilde{x},\overline{x})}\left(f_i(\widetilde{x})-f_i(\overline{x})\right)-\sum_{j=1}^{m}\frac{v_j^0}{\gamma_j(\widetilde{x},\overline{x})}h_j(\overline{x}) \geqq$$
$$\sum_{i=1}^{p}\lambda_i^0\widehat{f}_i(\overline{x},\eta(\widetilde{x},\overline{x}))+\sum_{j=1}^{m}v_j^0\widehat{h}_j(\overline{x},\eta(\widetilde{x},\overline{x}))+$$
$$\left(\sum_{i=1}^{p}\frac{\lambda_i^0\rho_i^1}{\alpha_i(\widetilde{x},\overline{x})}+\sum_{j=1}^{m}\frac{v_j^0\rho_j^3}{\gamma_j(\widetilde{x},\overline{x})}\right)d(\widetilde{x},\overline{x}) \geqq 0,$$

where the last inequality is according to (3.1), $d(\widetilde{x},\overline{x}) \geqq 0$, and $\sum_{i=1}^{p}\frac{\lambda_i^0\rho_i^1}{\alpha_i(\widetilde{x},\overline{x})}+\sum_{j=1}^{m}\frac{v_j^0\rho_j^3}{\gamma_j(\widetilde{x},\overline{x})} \geqq 0$. Hence,

$$\sum_{i=1}^{p}\frac{\lambda_i^0}{\alpha_i(\widetilde{x},\overline{x})}\left(f_i(\widetilde{x})-f_i(\overline{x})\right)-\sum_{j=1}^{m}\frac{v_j^0}{\gamma_j(\widetilde{x},\overline{x})}h_j(\overline{x}) \geqq 0. \tag{3.10}$$

By (3.3), (3.5) and $\overline{x}\in X$ we get $v_j^0h_j(\overline{x})=0,\forall j\in M$. Therefore, from (3.10), we obtain

$$\sum_{i=1}^{p}\frac{\lambda_i^0}{\alpha_i(\widetilde{x},\overline{x})}\left(f_i(\widetilde{x})-f_i(\overline{x})\right) \geqq 0. \tag{3.11}$$

Using (3.4), $\alpha_i\left(\widetilde{x},\overline{x}\right)>0,\forall\ i\in P$ and (3.11), we obtain that there exists $i_0\in P$ such that

$$f_{i_0}(\widetilde{x}) \geqq f_{i_0}(\overline{x}). \tag{3.12}$$

By (3.2), (3.8), $\beta_i\left(\widetilde{x},\overline{x}\right)>0,\forall i\in P$, $d(\widetilde{x},\overline{x}) \geqq 0$, and $\rho^2 \geqq 0$ it follows

$$g_i(\widetilde{x}) \leqq g_i(\overline{x}),\forall\ i\in P. \tag{3.13}$$

Now, using (3.12), (3.13), $f(\overline{x}) \geqq 0$ and $g>0$, we obtain

$$\frac{f_{i_0}(\widetilde{x})}{g_{i_0}(\widetilde{x})} \geqq \frac{f_{i_0}(\overline{x})}{g_{i_0}(\overline{x})},$$

which is in contradiction to (3.6). □

Theorem 3.2. *Let* $\overline{x} \in X$, $u_i^0 = \dfrac{f_i(\overline{x})}{g_i(\overline{x})}, \forall i \in P$, *and* (f, g, h) *be of general* $\left(\rho^1, \rho^2, \rho^3, d\right)$ *- type I at* $\overline{x}$. *Also, we assume that there exists* $\lambda^0 \in \mathbf{R}^p, v^0 \in \mathbf{R}^m$ *such that* $\lambda^{0^T}\rho^1 - \sum_{i=1}^{p} \lambda_i^0 u_i^0 \rho_i^2 + v^{0^T}\rho^3 \geqq 0$,

$$\sum_{i=1}^{p} \lambda_i^0 \left(\alpha_i\left(\widetilde{x}, \overline{x}\right) \widehat{f}_i(\overline{x}, \eta(x, \overline{x})) - u_i^0 \beta_i\left(\widetilde{x}, \overline{x}\right) \widehat{g}_i(\overline{x}, \eta(x, \overline{x}))\right) + \sum_{j=1}^{m} v_j^0 \gamma_j\left(\widetilde{x}, \overline{x}\right) \widehat{h}_j(\overline{x}, \eta(x, \overline{x})) \geqq 0, \forall x \in X, \quad (3.14)$$

$$v^{0^T} h(\overline{x}) = 0, \quad (3.15)$$

$$\lambda^0 \geq 0, \quad (3.16)$$

$$u^0 \geqq 0, \quad (3.17)$$

$$v^0 \geqq 0. \quad (3.18)$$

Then $\overline{x}$ *is a weak minimum solution for (VFP).*

Proof: We proceed by contradicting. If $\overline{x}$ is not a weak minimum solution for (VFP), there exists $\widetilde{x} \in X$ such that

$$\frac{f_i(\widetilde{x})}{g_i(\widetilde{x})} < \frac{f_i(\overline{x})}{g_i(\overline{x})}, \forall\, i \in P,$$

i.e.,

$$f_i(\widetilde{x}) < u_i^0 g_i(\widetilde{x}), \forall i \in P. \quad (3.19)$$

Since (f, g, h) is of general $\left(\rho^1, \rho^2, \rho^3, d\right)$ - type I at $\overline{x}$, we get

$$f_i(\widetilde{x}) - f_i(\overline{x}) \geqq \alpha_i\left(\widetilde{x}, \overline{x}\right) \widehat{f}_i(\overline{x}, \eta(\widetilde{x}, \overline{x})) + \rho_i^1 d(\widetilde{x}, \overline{x}),\ i \in P,$$
$$g_i(\widetilde{x}) - g_i(\overline{x}) \leqq \beta_i\left(\widetilde{x}, \overline{x}\right) \widehat{g}_i(\overline{x}, \eta(\widetilde{x}, \overline{x})) - \rho_i^2 d(\widetilde{x}, \overline{x}),\ i \in P,$$
$$-h_j(\overline{x}) \geqq \gamma_j\left(\widetilde{x}, \overline{x}\right) \widehat{h}_j(\overline{x}, \eta(\widetilde{x}, \overline{x})) + \rho_j^3 d(\widetilde{x}, \overline{x}),\ j \in M.$$

Using these inequalities, (3.16), (3.17), and (3.18), we get

$$\sum_{i=1}^{p} \lambda_i^0 \left(f_i(\widetilde{x}) - f_i(\overline{x})\right) - \sum_{i=1}^{p} \lambda_i^0 u_i^0 \left(g_i(\widetilde{x}) - g_i(\overline{x})\right) - \sum_{j=1}^{m} v_j^0 h_j(\overline{x}) \geqq$$
$$\sum_{i=1}^{p} \lambda_i^0 \left(\alpha_i\left(\widetilde{x}, \overline{x}\right) \widehat{f}_i(\overline{x}, \eta(\widetilde{x}, \overline{x})) - u_i^0 \beta_i\left(\widetilde{x}, \overline{x}\right) \widehat{g}_i(\overline{x}, \eta(\widetilde{x}, \overline{x}))\right) +$$
$$\sum_{j=1}^{m} v_j^0 \gamma_j\left(\widetilde{x}, \overline{x}\right) \widehat{h}_j(\overline{x}, \eta(\widetilde{x}, \overline{x})) + \left(\lambda^{0^T}\rho^1 - \sum_{i=1}^{p} \lambda_i^0 u_i^0 \rho_i^2 + v^{0^T}\rho^3\right) d(\widetilde{x}, \overline{x}) \geqq 0,$$

where the last inequality is according to (3.14), $d(\widetilde{x}, \overline{x}) \geqq 0$, and $\lambda^{0^T}\rho^1 - \sum_{i=1}^{p} \lambda_i^0 u_i^0 \rho_i^2 +$

${v^0}^T \rho^3 \geqq 0$. Therefore,

$$\sum_{i=1}^{p} \lambda_i^0 \left[\left(f_i(\widetilde{x}) - u_i^0 g_i(\widetilde{x}) \right) - \left(f_i(\overline{x}) - u_i^0 g_i(\overline{x}) \right) \right] - \sum_{j=1}^{m} v_j^0 h_j(\overline{x}) \geqq 0.$$

Since $u_i^0 = \dfrac{f_i(\overline{x})}{g_i(\overline{x})}, \forall i \in P$, we obtain

$$\sum_{i=1}^{p} \lambda_i^0 \left(f_i(\widetilde{x}) - u_i^0 g_i(\widetilde{x}) \right) - {v^0}^T h(\overline{x}) \geqq 0.$$

Now, (3.15) gives

$$\sum_{i=1}^{p} \lambda_i^0 \left(f_i(\widetilde{x}) - u_i^0 g_i(\widetilde{x}) \right) \geqq 0. \tag{3.20}$$

From (3.16), we get that there exists $i_0 \in P$ such that:

$$f_{i_0}(\widetilde{x}) - u_{i_0}^0 g_{i_0}(\widetilde{x}) \geqq 0,$$

which contradicts (3.19). ∎

4 Duality

Let $\widehat{f}_i, \widehat{g}_i, (i \in P), \widehat{h}_j (j \in M), \rho^1, \rho^2, \rho^3, \eta$, and d be as in Section 2. We consider, for (VFP), a general Mond-Weir dual (FMWD) as

$$\max \psi(y, \lambda, u, v) = u - v_{I_0}^T h_{I_0}(y) e$$

subject to:

$$\sum_{i=1}^{p} \lambda_i \left(\widehat{f}_i(y, \eta(x, y)) - u_i \widehat{g}_i(y, \eta(x, y)) \right) + \sum_{j=1}^{m} v_i \widehat{h}_i(y, \eta(x, y)) \geqq 0, \tag{4.1}$$

for all $x \in X$,

$$f_i(y) - u_i g_i(y) \geqq 0 \text{ for any } i \in P, \tag{4.2}$$

$$v_{I_s}^T h_{I_s}(y) \geqq 0 \ (1 \leqq s \leqq \gamma), \tag{4.3}$$

$$\lambda^T e = 1, \ \lambda \geq 0, \ \lambda \in \mathbf{R}^p, \tag{4.4}$$

$$u \geqq 0, \ u \in \mathbf{R}^p, v \geqq 0, \ y \in X_0, \tag{4.5}$$

where $\gamma \geqq 1$, $I_s \cap I_t = \emptyset$ for $s \neq t$ and $\bigcup_{s=0}^{\gamma} I_s = M$. (Here $v_{I_s} = (v_j)_{j\in I_s}$, $h_{I_s} = (h_j)_{j\in I_s}$).

Let W denote the set of all feasible solutions of (FMWD). Also, we define the following sets

$$A = \{(\lambda, u, v) \in \mathbf{R}^p \times \mathbf{R}^p \times \mathbf{R}^m | (y, \lambda, u, v) \in W \text{ for some } y \in X_0\}$$

and, for $(\lambda, u, v) \in A$,

$$B(\lambda, u, v) = \{y \in X_0 \mid (y, \lambda, u, v) \in W\}.$$

We put $B = \bigcup_{(\lambda,u,v)\in A} B(\lambda, u, v)$ and note that $B \subset X_0$. Also, we note that if $(y, \lambda, u, v) \in W$ then $(\lambda, u, v) \in A$ and $y \in B(\lambda, u, v)$.

Theorem 4.1. *(Weak Duality). Assume that for all feasible solutions $x \in X$ and $(y, \lambda, u, v) \in W$ for (VFP) and (FMWD) respectively, we have*

$$\widehat{f}_i\left(y, \eta\left(x, y\right)\right) - u_i \widehat{g}_i\left(y, \eta\left(x, y\right)\right) + \sum_{j\in I_0} v_j \widehat{h}_j\left(y, \eta\left(x, y\right)\right) \geqq -\rho_i^1 d\left(x, y\right) \Rightarrow f_i\left(x\right) - u_i g_i\left(x\right) + v_{I_0}^T h_{I_0}\left(x\right) \geq f_i\left(y\right) - u_i g_i\left(y\right) + v_{I_0}^T h_{I_0}\left(y\right),$$

for all $i \in P$, *(4.6)*

and

$$-v_{I_s}^T h_{I_s}\left(y\right) \leqq 0 \Rightarrow \sum_{j\in I_s} v_j \widehat{h}_j\left(y, \eta\left(x, y\right)\right) \leqq -\rho_s^3 d\left(x, y\right),$$

for $1 \leqq s \leqq \gamma$, *(4.7)*

hold on $B(\lambda, u, v)$ and $\sum_{i=1}^{p} \lambda_i \rho_i^1 + \sum_{s=1}^{\gamma} \rho_s^3 \geqq 0$. Then the following cannot hold:

$$f_i(x) - u_i g_i(x) \leqq v_{I_0}^T h_{I_0}(y), \text{ for any } i \in P, \tag{4.8}$$

and

$$f_{i_0}(x) - u_{i_0} g_{i_0}(x) < v_{I_0}^T h_{I_0}(y), \text{ for some } i_0 \in P. \tag{4.9}$$

Proof: Using (4.3) and (4.7), we obtain

$$\sum_{j\in I_s} v_j \widehat{h}_j\left(y, \eta\left(x, y\right)\right) \leqq -\rho_s^3 d\left(x, y\right),\ 1 \leqq s \leqq \gamma. \tag{4.10}$$

Now we suppose to the contrary of the result of the theorem that (4.8) and (4.9) hold. Hence if (4.8) and (4.9) hold for some feasible x for (VFP) and (y, λ, u, v) feasible for (FMWD), we obtain

$$f_i(x) - u_i g_i(x) \leqq v_{I_0}^T h_{I_0}(y), \text{ for any } i \in P \tag{4.11}$$

and

$$f_{i_0}(x) - u_{i_0} g_{i_0}(x) < v_{I_0}^T h_{I_0}(y), \text{ for some } i_0 \in P. \tag{4.12}$$

According to (4.2), (4.5) and the feasibility of x for (VFP), we have

$$v_{I_0}^T h_{I_0}(x) \leqq 0 \leqq f_i(y) - u_i g_i(y), \text{ for all } i \in P. \tag{4.13}$$

Combining (4.11)-(4.13) we get

$$f_i(x) - u_i g_i(x) + v_{I_0}^T h_{I_0}(x) \leqq f_i(y) - u_i g_i(y) + v_{I_0}^T h_{I_0}(y), \forall i \in P, \tag{4.14}$$

and

$$f_{i_0}(x) - u_{i_0} g_{i_0}(x) + v_{I_0}^T h_{I_0}(x) < f_{i_0}(y) - u_{i_0} g_{i_0}(y) + v_{I_0}^T h_{I_0}(y),$$

for some $i_0 \in P$. (4.15)

By (4.6), (4.14) and (4.15) we obtain

$$\widehat{f}_i(y, \eta(x,y)) - u_i \widehat{g}_i(y, \eta(x,y)) + \sum_{j \in I_0} v_j \widehat{h}_j(y, \eta(x,y))$$

$$< -\rho_i^1 d(x,y), \text{ for any } i \in P. \tag{4.16}$$

By (4.4) and (4.16) we get

$$\sum_{i=1}^{p} \lambda_i \left(\widehat{f}_i(y, \eta(x,y)) - u_i \widehat{g}_i(y, \eta(x,y)) \right) + \sum_{j \in I_0} v_j \widehat{h}_j(y, \eta(x,y)) < -\sum_{i=1}^{p} \lambda_i \rho_i^1 d(x,y).$$

Now, by (4.1) and $\sum_{i=1}^{p} \lambda_i \rho_i^1 + \sum_{s=1}^{\gamma} \rho_s^3 \geqq 0$ we obtain

$$\sum_{s=1}^{\gamma} \sum_{j \in I_s} v_j \widehat{h}_j(y, \eta(x,y)) > \sum_{i=1}^{p} \lambda_i \rho_i^1 d(x,y) \geqq -\sum_{s=1}^{\gamma} \rho_s^3 d(x,y),$$

which is a contradiction to (4.10). Thus the theorem is proved. □

References

[1] T. Antczak, (p,r)-invex sets and functions, J. Math. Anal. Appl. 263 (2001) 355-379.

[2] B. D. Corley, Duality theory for maximization with respect to cones, J. Math. Anal. Appl. 84 (1981) 560-568.

[3] R. R. Egudo, Efficiency and generalized convex duality for multiobjective programs, J. Math. Anal. Appl. 138 (1989) 84-94.

[4] K. H. Elster, R. Nehse, Optimality Conditions for Some Nonconvex Problems, Springer Verlag, New York, 1980.

[5] A. M. Geoffrion, Proper efficiency and the theory of vector maximization, J. Math. Anal. Appl.22 (1968) 618-630.

[6] M. Hayashi, H. Komiya, Perfect duality for convexlike programs, J. Optim. Theory Appl. 38 (1982) 179-189.

[7] V. Jeyakumar, Equivallence of saddle-points and optima for and duality for a class of non-smooth non-convex problems, J. Math. Anal. Appl. 130 (1988) 334-343.

[8] S. K. Mishra, Generalized proper efficiency and duality for a class of non-differentiable multiobjective variational problems with V-invexity, J. Math. Anal. Appl. 202 (1996) 53-71.

[9] S.K. Mishra, Lagrange multipliers saddle points and scalarizations in composite multiobjective nonsmooth programming, Optimization 38 (1996), 93-105.

[10] S. K. Mishra, On multiple-objective optimization with generalized univexity, J. Math. Anal. Appl. 224 (1998) 131-148.

[11] S.K. Mishra and R.N. Mukherjee, Generalized convex composite multiobjective nonsmooth programming and conditional proper efficiency, Optimization 34 (1995), 53–66.

[12] S. K. Mishra, R. N. Mukherjee, Multiobjective control problems with V-invexity, J. Math. Anal. Appl. 235 (1999) 1-12.

[13] S. K. Mishra, R. N. Mukherjee, On generalized convex multiobjective nonsmooth programming, J. Austral. Math. Soc., Series B-38, 140-148.

[14] S. K. Mishra, M. A. Noor, Some nondifferentiable multiobjective programming problems, J. Math. Anal. Appl. in press.

[15] S. K. Mishra, S. Y. Wang, K. K. Lai, Multiple objective fractional programming involving semilocally type I-preinvex and related functions, J. Math. Anal. Appl. 310 (2005) 626-640.

[16] S. Mititelu, Invex sets, Math. Rep. 46 (1994) 529-532.

[17] R. N. Mukherjee, S. K. Mishra, Multiobjective programming with semilocally convex functions, J. Math. Anal. Appl. 199 (1996) 409-424.

[18] C. Niculescu, Optimality and duality in multiobjective fractional programming involving ρ-semilocally type I-preinvex and related functions, J. Math. Anal. Appl. 335 (2007) 7–19.

[19] V. Preda, Optimality conditions and duality in multiple objective programming involving semilocally convex and related functions, Optimization 36 (1996) 219-230.

[20] V. Preda, Optimality and duality in fractional multiple objective programming involving semilocally preinvex and related functions, J. Math. Anal. Appl. 288 (2003) 365-382.

[21] V. Preda, On Optimality and Duality in Nondifferentiable Multiobjective Optimization Involving Generalized d-Type-I and Related Functions, preprint, 2007.

[22] V. Preda, I. M. Stancu-Minasian, Duality in multiple objective programming involving semilocally preinvex and related functions, Glas. Mat. 32 (1997) 153-165.

[23] V. Preda, I. M. Stancu-Minasian, A. Batatorescu, Optimality and duality in nonlinear programming involving semilocally preinvex and related functions, J. Inf. Optim. Sci. 17 (1996) 585–596.

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